BASIC ELECTRICAL ENGINEERING

UNIT-2

SINGLE PHASE AC

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google: reactance vs resistance

Terms and Definitions

1) Waveform: graphical representation of alternating quantity

2) Cycle: complete set of all possible values of alternating quantity

3) Amplitude: peak value / max. possible value taken in a cycle

4) Peak to peak value: magnitude of values between maxima in +ve to -ve cycle

5) Time period (T): time taken to complete one cycle

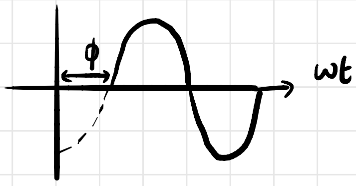
6) Frequency (f): no of cycles in one time period

7) Angular frequency (ω): no. of circular cycles in one time period

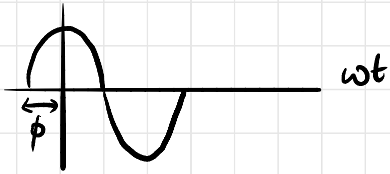
8) Phase (t): fraction of time period that has elapsed since the reference chosen.

9) Phase angle (ϕ or θ): fraction of 2π that alternating quantity has revolved by in rotational analog, with reference ($\theta = \omega t$)

10) Phase shift: phase that waveform leads / lags by reference

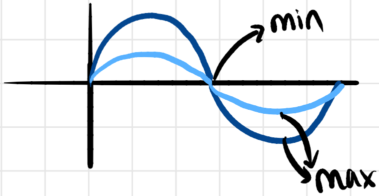


lag: $i = I_m \sin(\omega t - \phi)$

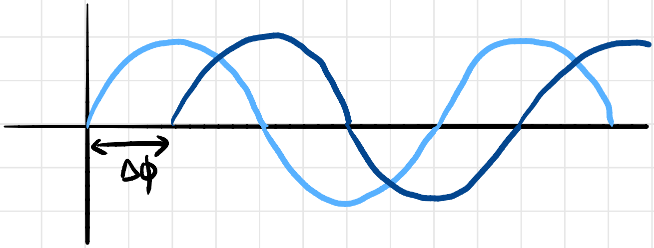


lead: $i = I_m \sin(\omega t + \phi)$

11) In phase: If two alternating quantities reach max and min at same instant of time, they are said to be in phase ($\Delta\phi = 0$)

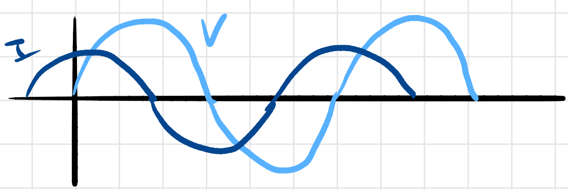


12) Phase difference ($\Delta\phi$): If they reach min/max at diff. times.



13) Out of phase: If one alternating quantity reaches +ve maximum when the other reaches -ve max, they are out of phase ($\Delta\phi = 180°$ or π)

14) Phase lead: If an alternating quantity achieves maximum before the reference quantity reaches maximum, it is leading



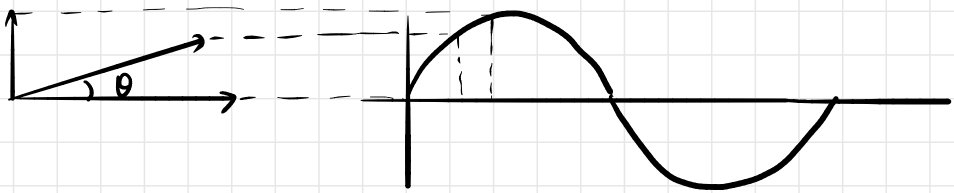
I leads V

15) Phase lag: one quantity reaches max. after reference quantity reaches

in prev. diagram, V lags I

16) Phasor: It is a moving/rotating vector. Alternate way of representing the alternating quantity. Represents magnitude and phase of alternating quantity.

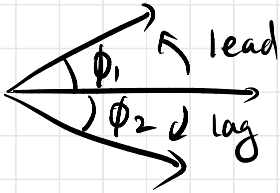
Phasor representation



17) Phasor diagram: It represents phase relationship between two quantities (rms value represented in phasor)

18) RMS value: Effective value of DC (current/voltage) which gives same heat as AC in one cycle. Mathematically root mean square.

19) Lead-lag in phasor:



20) Average value & rms value

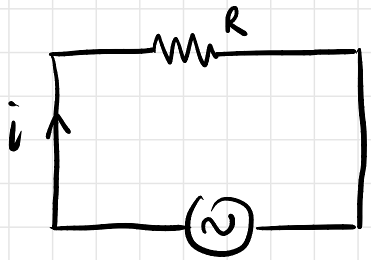
Average in full cycle = 0

$$\left(\frac{1}{\pi} = \frac{1}{\int_0^{\pi} d\omega t}\right)$$

Average in half cycle = $I_{avg} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\omega t \, d\omega t = \frac{2I_m}{\pi}$

rms value in full cycle: $I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2\omega t \, d\omega t} = \frac{I_m}{\sqrt{2}}$

Response of Pure R in AC Circuits



$v = V_m \sin \omega t$

interested in
- voltage
- current
- phase relationship
- power

- capital letters: rms value
- small letters: instantaneous value

According to Ohm's Law

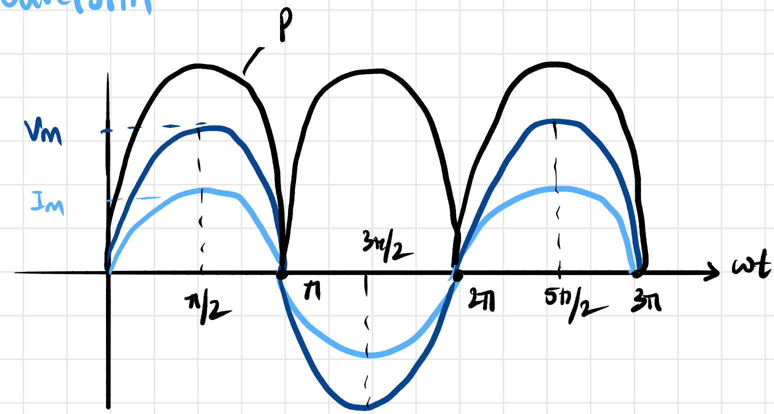
$$i = \frac{v}{R}$$

$$i = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$$

$$\boxed{i = I_m \sin \omega t}$$

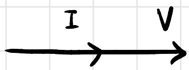
- Voltage & current are in phase
- Phase angle $\phi = 0^\circ$

Waveform



Phasor Diagram

- always rms value



Average power

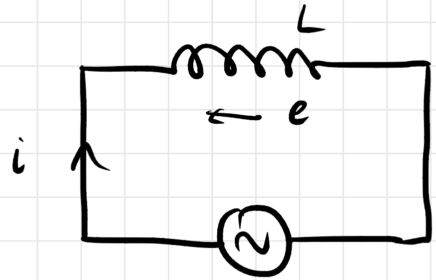
$$P = \frac{1}{2\pi}\int_0^{2\pi} vi \, d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m I_m \sin^2\omega t \, d\omega t$$

$$= \frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{1-\cos 2\omega t}{2} = \frac{V_m I_m}{2\pi}\left(\frac{2\pi}{2} - 0\right)$$

$$\boxed{P = \frac{V_m I_m}{2}} = V_{rms} I_{rms}$$

Response of Pure L in AC Circuits



$v = V_m \sin \omega t$

According to KVL (Actually Faraday's Law/ Lenz's law)

$$v = -e$$

$$e = -L\frac{di}{dt}$$

$$v = L\frac{di}{dt}$$

$$\int di = \int \frac{v\,dt}{L}$$

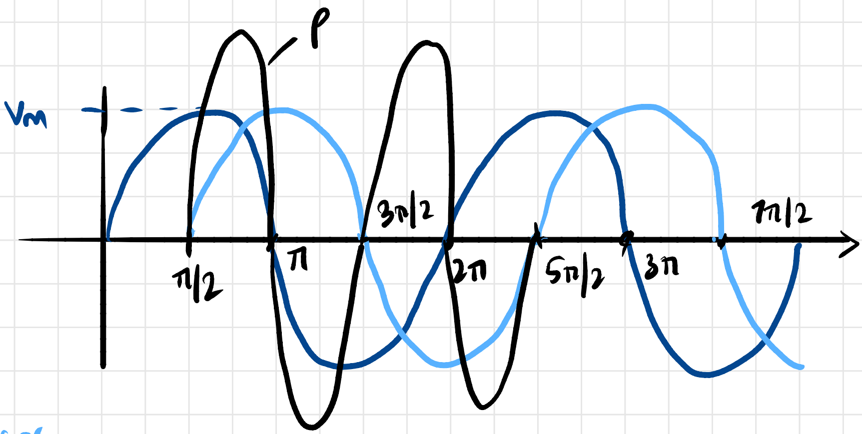
$$i = \int \frac{V_m \sin \omega t\,dt}{L} = \frac{V_m}{L}\left[\frac{-\cos \omega t}{\omega}\right]$$

$$i = \frac{V_m}{\omega L}(-\cos \omega t) = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)$$

$$i = \frac{V_m}{X_L}\sin(\omega t - \pi/2)$$

- Phase lag $= \pi/2$

Waveform



Phasor

Average Power

$$P = \frac{1}{2\pi}\int_0^{2\pi} vi \, d\omega t = \frac{V_m I_m}{2\pi}\int_0^{2\pi} -\sin\omega t \cos\omega t \, dt$$

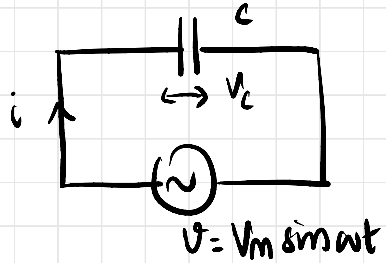
$$= \frac{V_m I_m}{2\pi}\left[\frac{\cos^2\omega t}{2}\right]_0^{2\pi} = 0 \qquad P = 0$$

- No power dissipated in an ideal inductor.

Impedence $Z = \frac{V \angle 0^\circ}{I \angle -90^\circ} = Z \angle 90^\circ$

$Z = jX_L$

Response of Pure C in AC Circuits



According to KVL

$$v - V_C = 0$$

$$v - \frac{q}{C} = 0$$

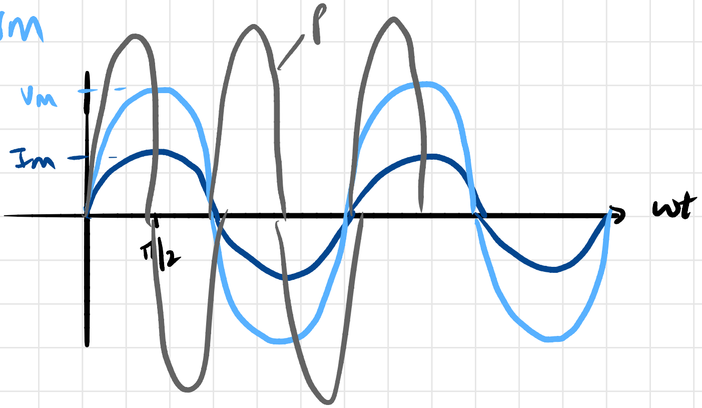
$$q = C(V_m \sin\omega t)$$

$$i = \frac{dq}{dt} = C\omega V_m \cos\omega t$$

$$i = \frac{V_m}{X_C} \sin(\omega t + \pi/2)$$

$$i = I_m \sin(\omega t + \pi/2) \qquad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Waveform



Average power

$$P = \frac{1}{2\pi}\int_0^{2\pi}(V_m \sin\omega t)(I_m \cos\omega t)\, d\omega t$$

$$= \frac{V_m I_m}{2\pi}\left[\frac{\sin^2\omega t}{2}\right]_0^{2\pi} = 0$$

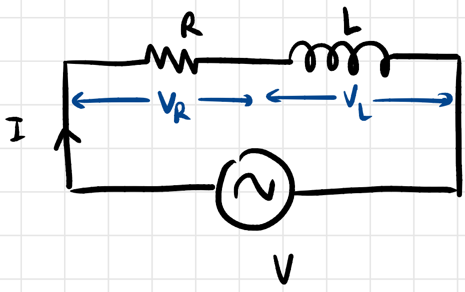
$P = 0$

- No power dissipated.

$Z = -jX_c$

SERIES CIRCUITS

1. Series RL Circuit



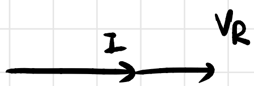
- capital letters are rms values

$$\bar{V} = \bar{V}_R + \bar{V}_L$$ (phasor sum / vector sum)

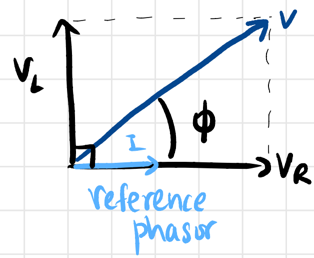
$$V = \sqrt{V_R^2 + V_L^2 + 2V_R V_L \cos\theta}$$

Phasor Diagram

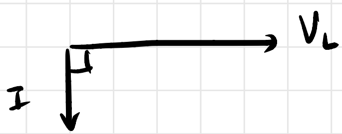
resistor



combination



inductor



$$V = \sqrt{V_L^2 + V_R^2}$$

- V leads I by some angle ϕ
- Applied voltage and current are out of phase

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2}$$

$\boxed{V = IZ}$ → Z is impedence

$$Z = \sqrt{R^2 + X_L^2} \quad \text{(unit: ohms)}$$

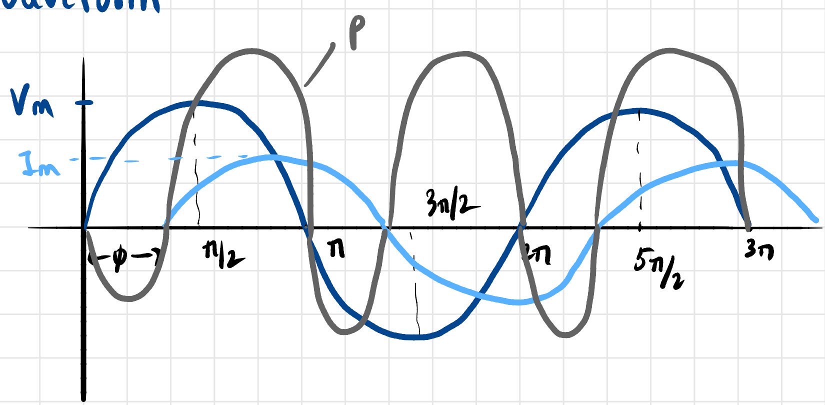
$$Z = R + jX_L \quad \text{(cartesian)} \qquad Z \angle \phi \quad \text{(polar)}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t - \phi)$$

Waveform



Average power

$$\langle P \rangle = \frac{1}{2\pi}\int_0^{2\pi} v i \, d(\omega t) = \frac{V_m I_m}{4\pi}\int_0^{2\pi} 2\sin(\omega t)\sin(\omega t - \phi)\, d(\omega t)$$

$$\langle P \rangle = \frac{V_m I_m}{4\pi} \int_0^{2\pi} \cos(\phi) - \cos(2\omega t - \phi) \, d(\omega t)$$

$$= \frac{V_m I_m}{4\pi} \left(2\pi \cos\phi - \left(\frac{\sin(2\omega t - \phi)}{2} \right)_0^{2\pi} \right)$$

$$= \frac{V_m I_m}{2} \cos\phi - \frac{V_m I_m}{4\pi}(0)$$

$\langle P \rangle = V_{rms} I_{rms} \cos\phi \longrightarrow \phi : 0 \text{ to } \pi/2$

Power factor = $\cos\phi$

Active Power / True power / Real power (P)

actual power

- power due to i that is in phase with voltage
- work is done
- $V_{rms} I_{rms} \cos\phi = (IZ)(I)\left(\frac{R}{Z}\right) = I^2 R$
- Watts (W)

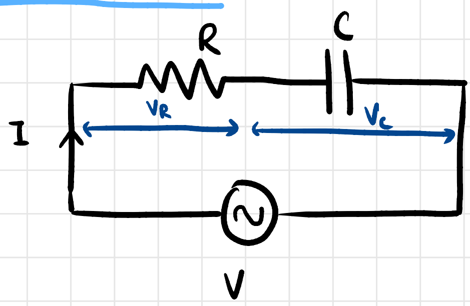
Reactive power (Q)

- $Q = VI \sin\phi$
- Var or VAR (Volt Ampere Reactive)

Apparent power (S)

- vectorial sum of $\bar{Q}$ and $\bar{P}$
- power supplied by source
- VA

2. Series RC circuit



$$\bar{V} = \bar{V}_R + \bar{V}_C$$

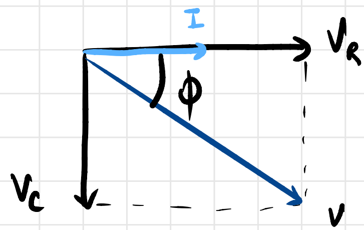
$$V = \sqrt{V_R^2 + V_C^2 +}$$

$$V = I\sqrt{R^2 + X_C^2}$$

$$V = V_m \sin \omega t$$

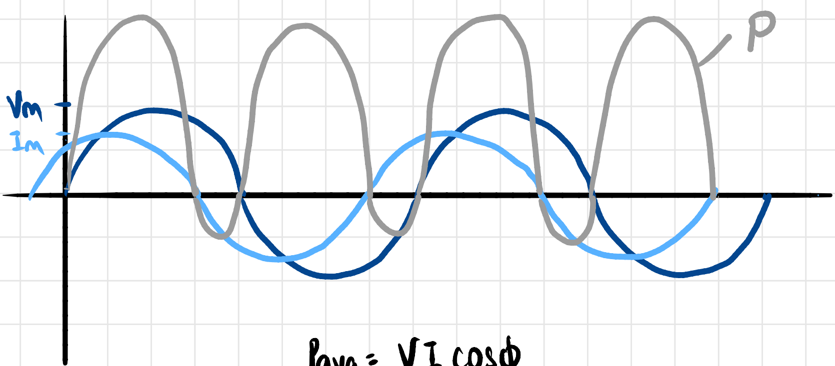
$$i = I_m \sin(\omega t + \phi)$$

Phasor



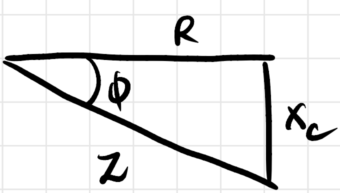
$$Z = R - jX_C$$

$$Z \angle -\phi$$

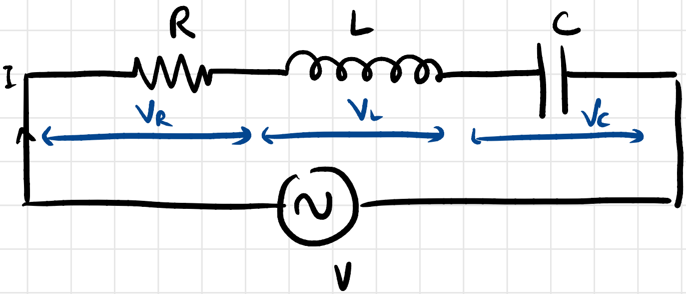


$$P_{avg} = VI\cos\phi$$

Impedance Triangle



3. Series RLC Circuit



(i) $V_L > V_C$

$$\overline{V} = \overline{V_R} + \overline{V_L - V_C}$$

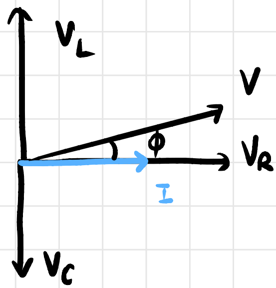
$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X = X_L - X_C$$

Phasor



$$Z = R + j(X_L - X_C)$$
$$= R + jX$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

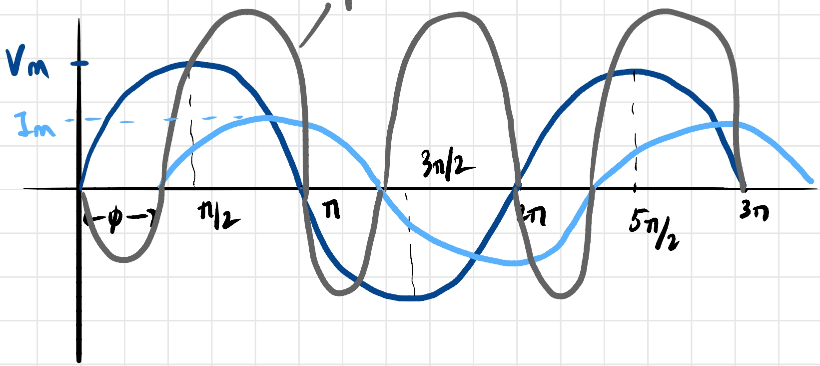
circuit is predominantly inductive

- current lags by ϕ

Power factor

- lagging power factor
- $\cos\phi = \frac{R}{Z}$

Waveform



(2) $V_C > V_L$

$$\bar{V} = \bar{V_R} + \overline{V_C - V_L}$$

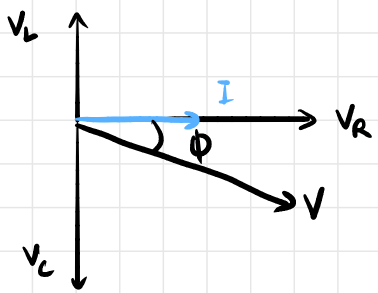
$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$V = I\sqrt{R^2 + (X_C - X_L)^2}$$

$$V = I\sqrt{R^2 + X^2}$$

$$v = V_m \sin \omega t$$
$$i = I_m \sin(\omega t + \phi)$$

Phasor



$$Z = R - jX$$

$$X = X_C - X_L$$

Power factor

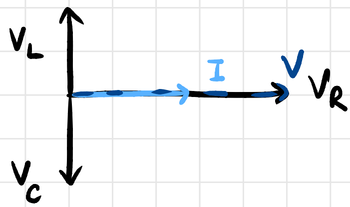
- leading power factor

(3) $V_L = V_C$

$$\vec{V} = \vec{V_R} + \overline{V_L - V_C}$$

$$V = V_R$$

Phasor



$Z = R$

Power factor

- $\cos\phi = 1$ ($\phi = 0$)
- $P = V_{rms} I_{rms}$
- current & voltage in phase

Note: if ϕ is +ve, current lags (mainly L)
if ϕ is −ve, current leads (mainly C)

Q1: A sinusoidal AC of $f = 25$ Hz has max value 100 A. How long will it take current to attain 20A, 50A and 100A?

$I_m = 100A$, $f = 25$ Hz, $\omega = 2\pi f = 50\pi = 157.08$

$i = I_m \sin \omega t$

$i = 100 \sin(50\pi t)$

(i) $20 = 100 \sin(50\pi t) \Rightarrow 50\cancel{\pi}t = \dfrac{11.537 \times \cancel{\pi}}{180}$

$t = 1.28$ ms

(ii) $50 = 100 \sin(50\pi t) \Rightarrow t = 3.33$ ms

(iii) $100 = 100 \sin(50\pi t) \Rightarrow t = 10$ ms

Q2. The max. values of alternating voltage and current are 400 V and 20 A respectively in a circuit connected to 50Hz supply. The instantaneous values of voltage and current are 238 V and 10 A respectively at time $t = 0$. Both increase positively.

(i) Write down the expression for voltage and current at time t.

(ii) Determine the power consumed in the circuit. Take current and voltage to be sinusoidal.

A: (always write units)

$v = 400 \sin(100\pi t + \phi_1)$

$i = 20 \sin(100\pi t + \phi_2)$

$238 = 400 \sin\phi_1 \Rightarrow \phi_1 = 0.637$ rad

$10 = 20 \sin\phi_2 \Rightarrow \phi_2 = \frac{\pi}{6}$ rad $= 0.523$

phase diff: (current lagging : mainly L)

$\phi = 0.11367$ rad $= 6.51^\circ$

$v = 400 \sin(100\pi t + 0.203\pi)$
$i = 20 \sin(100\pi t + \frac{\pi}{6})$

power consumed $= V_{rms} I_{rms} \cos\phi$

$= \frac{V_m I_m \cos\phi}{2} = \frac{400 \times 20 \times \cos 6.51}{2}$

$= 3974$ W (lagging)

Q3. A sinusoidal wave of $f = 50$ Hz has its max value of 9.2A. What will be its value at
(i) 2ms after the wave passes through 0 in +ve direction
(ii) 4.5 ms after the wave passes through +ve max.
Show the current values in a neat sketch of the waveform

A: $i = 9.2 \sin(100\pi t)$

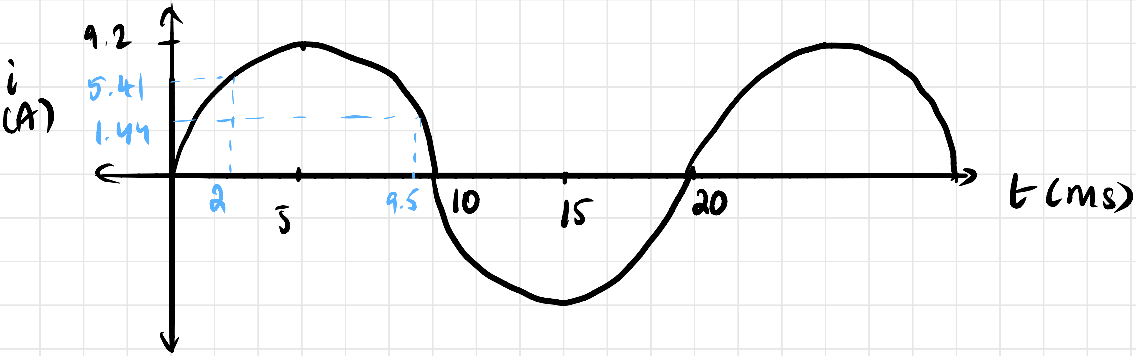
(i) $i = 9.2 \sin\left(\frac{100\pi \times 2}{1000}\right) = 5.41$ A

(ii) +ve max is at $\sin(100\pi t_1) = 1$

$100\pi t_1 = \frac{\pi}{2}$

$t_1 = \frac{1}{200}$ s $\Rightarrow t = \frac{1}{200} + \frac{4.5}{1000} = 9.5$ ms

$\therefore i = 9.2 \sin\left(\frac{100\pi \times 9.5}{1000}\right) = 1.44$ A



Q4. A sinusoidal voltage of 50 Hz has a max. value of $200\sqrt{2}$ V. At what time, measured from a positive max. value will the instantaneous voltage be equal to 141.4 V

$$v = 200\sqrt{2}\cos(100\pi t)$$

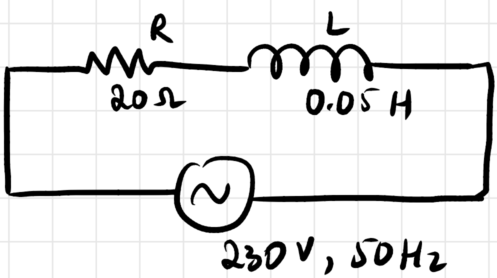
* do using sin

$$\frac{141.4}{200\sqrt{2}} = \cos(100\pi t)$$

$$100\pi t = \frac{\pi}{3} \Rightarrow t = 3.33 \text{ ms}$$

Q5. A circuit consists of $R = 20\,\Omega$ and $L = 0.05$ H connected in series. $V_{supply} = 230$ V, 50 Hz is applied across the circuit. Find current, power factor, power consumed by the circuit. Also draw the phasor diagram. (use complex)

$$v = 325.27 \sin(100\pi t)$$



$R = 20\,\Omega$, $\quad X_L = \omega L = 5\pi\,\Omega = 15.708\,\Omega$

$$Z = R + jX_L$$

$$I = \frac{V}{Z} = \frac{230}{20 + j(15.708)} = 7.11 - j5.59$$

$$I = 9.044, \quad \phi = -0.67\text{ rad} = -38.13^\circ$$

$$\cos\phi = \text{power factor} = 0.78 \text{ (lagging)}$$

$$\text{power} = VI\cos\phi = 230 \times 9.044 \times 0.78$$

$$= 1.63\text{ kW}$$

Q6. A series RC circuit having 4Ω resistance, $120\mu F$ capacitance connected across 230V, 50Hz supply. Calculate reactance, impedance (complex), current drawn (complex), power factor.

$C = 120 \times 10^{-6}\,F$
$R = 4\,\Omega$

$$X_C = -j\left(\frac{1}{2\pi fC}\right) = -26.53j$$

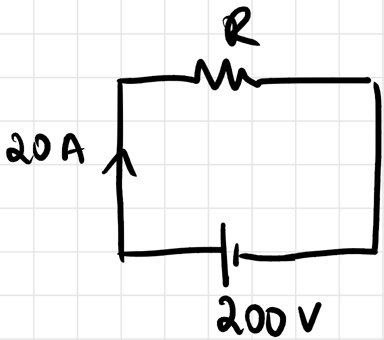
$R = 4$

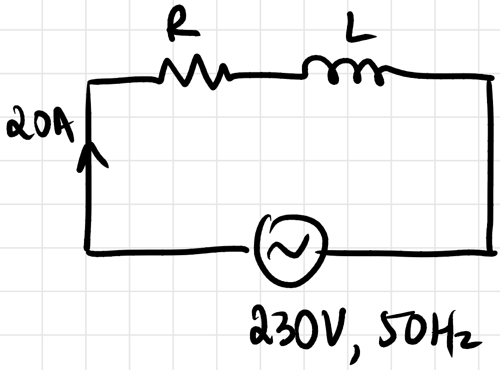
$$Z = 4 - 26.53j\ \Omega$$

$$I = \frac{V}{Z} = \frac{230}{4 - 26.53j} = 1.28 + 8.48j\text{ A} = 8.57\angle 81.4^\circ$$

$$\cos\phi = \cos(-81.42) = 0.149$$

Q7. A non-inductive load takes 20A, 200V. Calculate the inductance of the reactor to be connected in series in order that the same current be supplied from 230V, 50 Hz mains. Also determine phase angle between 230V supply and the current. Neglect resistance of reactor.





$R = 10\,\Omega$

$$Z = 10 + jX_L = 10 + j(100\pi L)$$

$$20 = \frac{230}{Z} \Rightarrow Z = 11.5$$

$$132.25 = 100 + (314L)^2$$

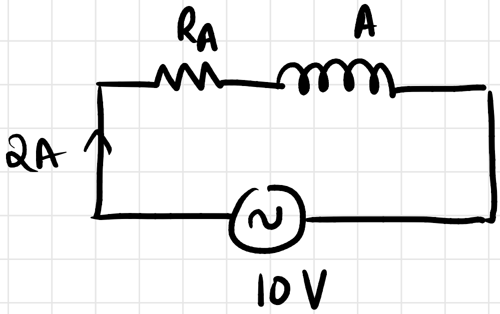
$$32.25 = 98696\,L^2$$

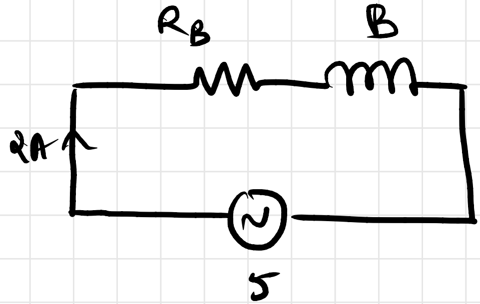
$$L = 18.07\text{ mH}$$

$$\tan\phi = \frac{X_L}{R} \Rightarrow \phi = 29.58°$$

current is lagging by 29.58°

Q8. Coil A takes 2A at a lagging PF of 0.8 with an applied P.D of 10V. Coil B takes 2A at a lagging PF of 0.7 with an applied voltage of 5V. What voltage will be required to produce a total current 2A with coil A and coil B in series. Find PF in this case.





$\cos\phi_A = 0.8$

$\phi_A = +36.87°$

$\cos\phi_B = 0.7$

$\phi_B = +45.57°$

$\frac{X_L}{R} = \frac{3}{4}$

$\frac{X_L}{R} = 1.02$

$Z_A = \frac{10}{2} = 5\angle 36.87$

$Z_B = 2.5\angle 45.57$

$Z_T = 5\angle 36.87 + 2.5\angle 45.57$

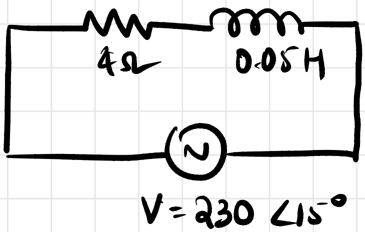
$Z_T = 7.48\angle 39.77$

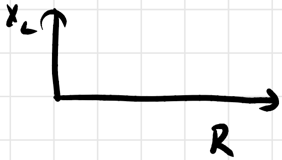
$V = 7.48 \times 2 = 14.96\ V$

$PF = 0.77$

current lags voltage by 39.77°

Q9 A sine voltage V= 230∠15°, f=50Hz, applied to series RL circuit, R=4Ω, L=0.09H. I_{rms}=? ϕ=? $\cos\phi$=? actual power=? reactive power=? Apparent power=?





$$X_L = 2\pi f L = 9\pi = 28.27$$

$$Z = 4 + j(9\pi)$$

$$I = \frac{V}{Z} = \frac{230\angle 15^\circ}{4 + j9\pi}$$

$$= 8.05\angle -66.95^\circ \text{ A} \quad (\text{wrt ref})$$

$$I_{\text{wrt } V} = 8.05\angle -81.95^\circ \text{ A} \quad (\text{wrt } V)$$

$\phi = 81.95^\circ$ (lagging current)

$\cos\phi = 0.14$ (lag)

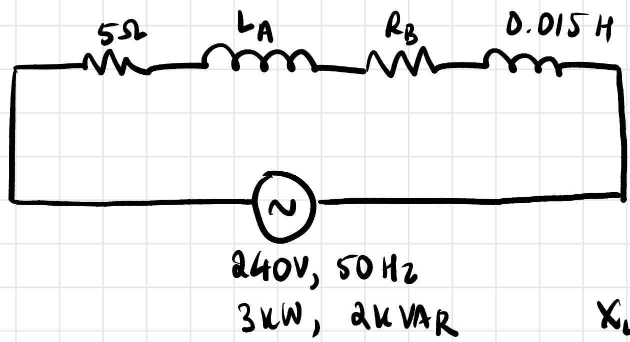
$P = VI\cos\phi = 259.21$ (I^2R)

$P = 259.21$ W

$Q = VI\sin\phi = 1833$ VAR (I^2X)

$S = VI = 1851.5$ VA (I^2Z)

Q10. Two coils A and B are connected in series across 240V, 50 Hz supply. The resistance of coil A is 5Ω and the inductance of coil B is 0.015 H. If the input from the supply is 3kW and 2kVAR, find the inductance of coil A and the resistance of coil B. Calculate V across each coil wrt I.



$X_{LB} = 4.71$

$P = 3kW = VI\cos\phi$
$Q = 2k\,VAR = VI\sin\phi$

$$\tan\phi = \frac{2}{3} \Rightarrow \phi = 33.69^\circ$$

$$2000 = 240 \times I \times \sin 33.69 \Rightarrow I = 15.02\ A$$

$$I = 15.02\angle -33.69^\circ$$

$$Z = \frac{V}{I} = \frac{240}{15.02\angle -33.69} = 13.29 + j\,8.86$$

$$13.29 = 5 + R_B \Rightarrow R_B = 8.29\,\Omega$$

$$8.86 = 2\pi f(0.015 + L_A) \Rightarrow L_A = 0.013\ H$$

$$X_{LA} = 4.15$$

$$Z_A = 5 + j(2\pi f L_A) = 5 + j\,4.08$$

$$V_A = I Z_A = (15.02)(5 + j\,4.15)$$

$$= 97.6 \angle 39.69^\circ$$

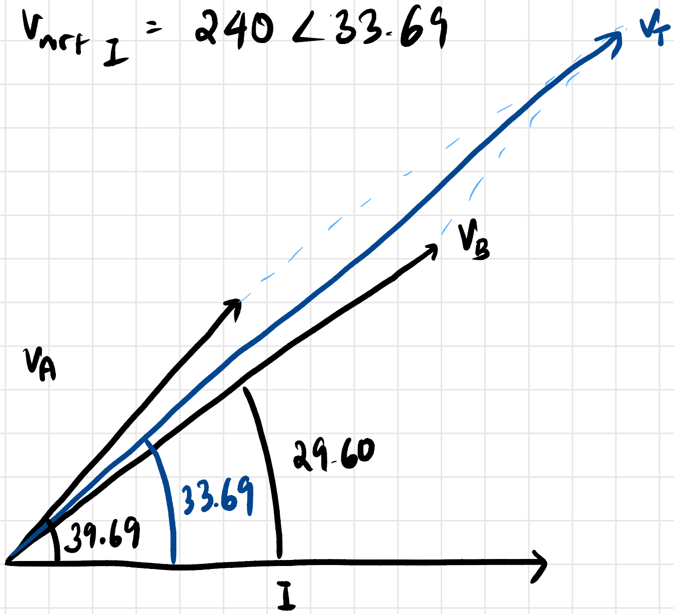
$$Z_B = 8.29 + j\,4.71$$

$$V_B = I Z_B = (15.02)(8.29 + j\,4.71)$$

$$= 143.21 \angle 29.60^\circ$$

$$V_{tot} = V_A + V_B = 239.91 \angle 33.69$$

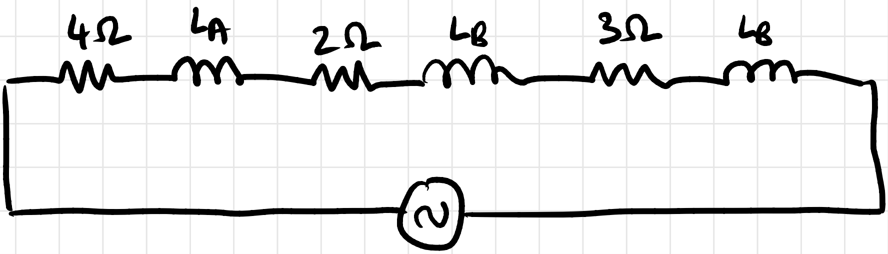
$$V_{ref\ I} = 240 \angle 33.69$$



Q11. Three coils A, B and C are connected in series. When a current of 3A is passed through the circuit, the voltage drops across the coils are 12V, 6V and 9V respectively on DC and 15V, 9V and 12V respectively on AC. Find each of the coils'

(i) internal parameters
(ii) power dissipated when AC flows through it
(iii) the AC applied voltage

Draw phasor diagram. Find overall P.F.



(i)

$$Z_A = \frac{15}{3} = 5\Omega \Rightarrow 5^2 = 4^2 + X_{LA}^2 \Rightarrow X_{LA} = 3\Omega$$
$$\theta_1 = 36.89^\circ$$

$$Z_B = \frac{9}{3} = 3\Omega \Rightarrow 3^2 = 2^2 + X_{LB}^2 \Rightarrow X_{LB} = 2.24\Omega$$
$$\theta_2 = 48.19^\circ$$

$$Z_C = \frac{12}{3} = 4\Omega \Rightarrow 4^2 = 3^2 + X_{LC}^2 \Rightarrow X_{LC} = 2.65\Omega$$
$$\theta_3 = 41.41^\circ$$

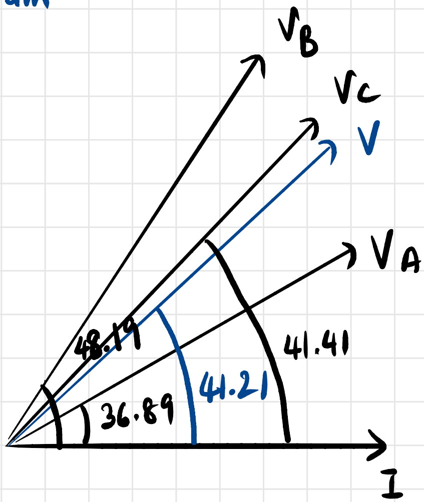
(ii) $P_A = 3^2 \times 4 = 36\,W$; $P_B = 3^2 \times 2 = 18\,W$; $P_C = 3^2 \times 3 = 27\,W$

$$P_{tot} = 81\,W$$

(iii) $Z_{tot} = (9 + j7.88) \Rightarrow V = IZ = 35.89 \angle 41.21\,V$

$$V = 35.89 \angle 41.21\,V$$

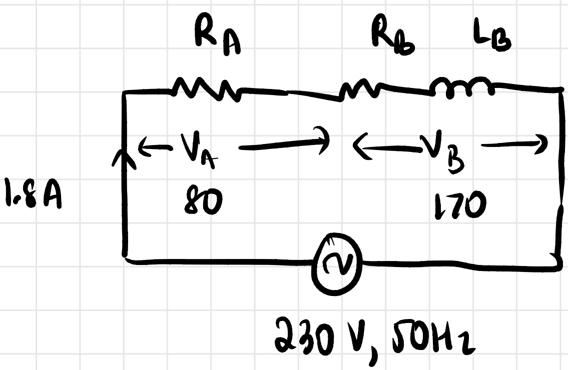
Phasor Diagram



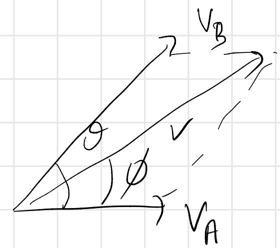
* note: do not represent Z values on phasor; only V values

power factor = $\cos\phi = 0.75$ lag

Q12. A non-inductive resistor is connected in series with a coil across 230V, 50Hz supply. The current is 1.8 A and the PD across the resistor and the coil are 80 V and 170 V respectively. Find the inductance and the resistance of the coil and the phase diff. b/w current and supply voltage and also draw the phasor diagram representing the circuit.



$$R_A = \frac{V_A}{1.8} = \frac{80}{1.8} = 44.44\ \Omega$$



$$Z_B = \frac{170}{1.8} = 94.44\ \Omega$$

$$\bar{V} = \bar{V}_A + \bar{V}_B \Rightarrow V^2 = V_A^2 + V_B^2 + 2V_A V_B \cos\theta$$

$$230^2 = 80^2 + 170^2 + 2 \times 80 \times 170 \cos\theta$$
$$\cos\theta = 11/17$$
$$\theta = 49.68^\circ$$

$$Z_B = 94.44 \angle 49.68^\circ$$
$$= 61.11 + j(72.08)$$

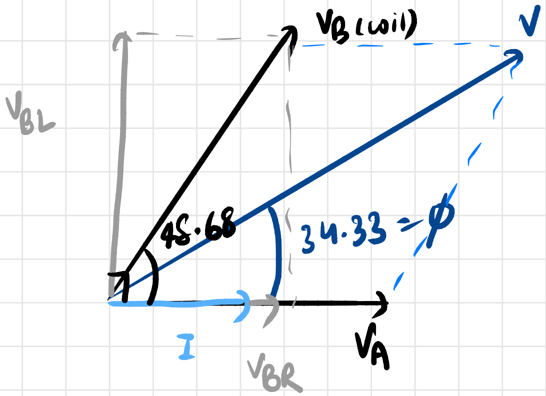
$$R_B = 61.11\ \Omega$$
$$X_{LB} = 72.08\ \Omega \Rightarrow L = \frac{X_L}{2\pi f} = 0.229\ \text{H}$$

$$Z_{tot} = 44.44 + 94.44 \angle 49.68$$
$$= 105.55 + 72.08j$$
$$= 127.81 \angle 34.33$$

$$\therefore \phi = 34.33^\circ$$

lagging pf

Phasor



PARALLEL CIRCUITS

we can have independent control over devices

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

conductance (℧)

$$G_T = G_1 + G_2 + \ldots$$

&

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots$$

admittance (℧)

$$Y_T = Y_1 + Y_2 + \ldots$$

$$\frac{1}{X_T} = \frac{1}{X_1} + \frac{1}{X_2} + \ldots$$

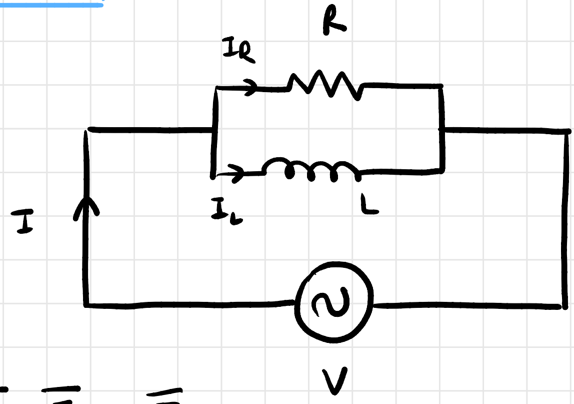
susceptance (℧)

$$B_T = B_1 + B_2 + \ldots$$

Admittance resolved into conductance and susceptance.

Admittance triangle

1. R-L Parallel



$$\bar{I} = \bar{I}_R + \bar{I}_L$$

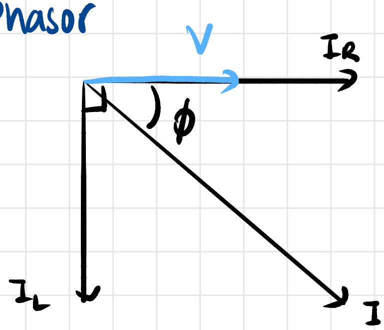
$$I = \sqrt{I_R^2 + I_L^2} = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L}\right)^2}$$

$$= V\sqrt{G^2 + B_L^2}$$

$$I = VY$$

$$Y = \sqrt{G^2 + B_L^2} = Y \angle -\phi \qquad Y = G - jB_L$$

Phasor



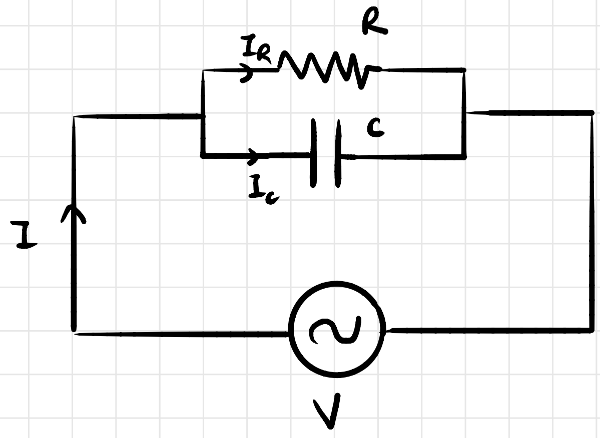
power factor = $\cos\phi$ lag

- current is always lagging in RL circuits.

$$v = V_m \sin \omega t$$

$$i = i_m \sin(\omega t - \phi) \quad \text{lagging}$$

2. R-C Parallel



$$\bar{I} = \bar{I}_R + \bar{I}_c$$

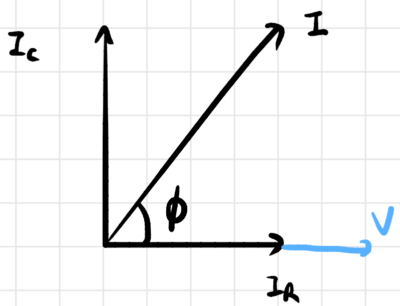
$$I = \sqrt{I_R^2 + I_c^2} = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_c}\right)^2}$$

$$I = V\sqrt{G^2 + B_c^2}$$

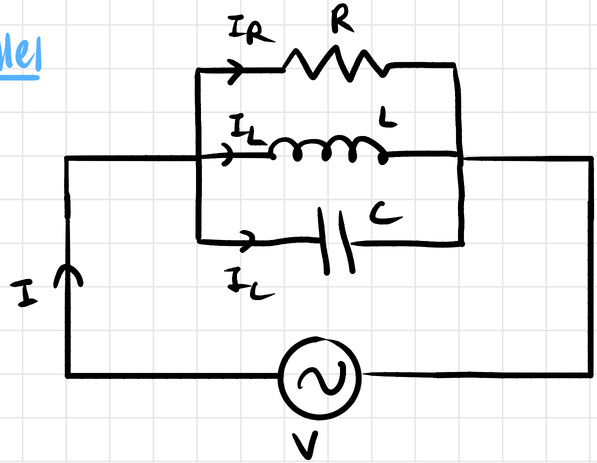
$$I = VY$$

$$Y = G + jB_c = Y\angle{+\phi}$$

Power factor = $\cos\phi$ lead



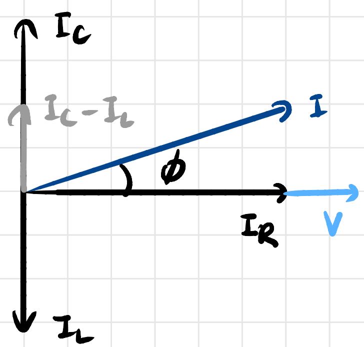
3. RLC Parallel



$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

Case I: $I_C > I_L$

Phasor



$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$= V\sqrt{G^2 + (B_C - B_L)^2}$$

$$Y = G + j(B_C - B_L)$$

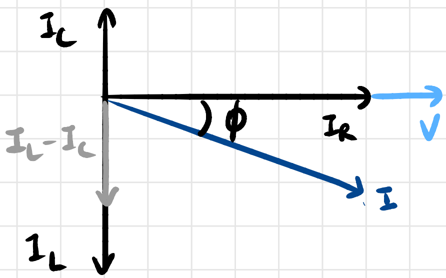
$$\cos\phi = \frac{B_C - B_L}{G}$$

$$v = V_m \sin\omega t$$

$$i = i_m \sin(\omega t + \phi)$$

Case II : $I_C < I_L$

Phasor



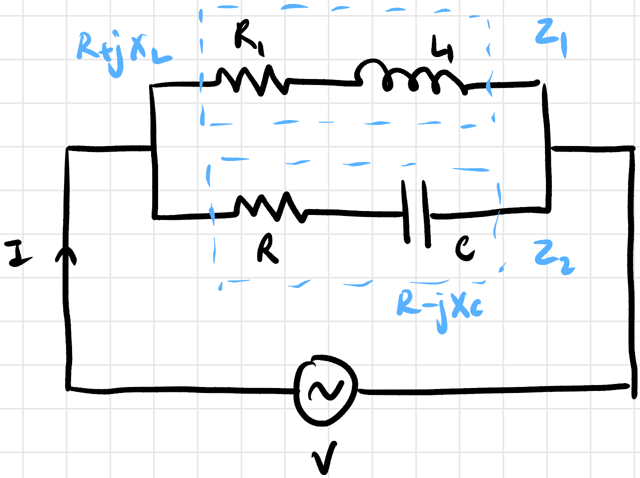
$$I = \sqrt{I_R^2 + (I_L - I_C)^2}$$
$$= V\sqrt{G^2 + (B_L - B_C)^2}$$
$$Y = G + j(B_L - B_C)$$

$$\cos\phi = \frac{B_L - B_C}{G}$$

$$v = V_m \sin\omega t$$
$$i = i_m \sin(\omega t - \phi)$$

Case III : $I_C = I_L$

Series and Parallel Combination



Impedance

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

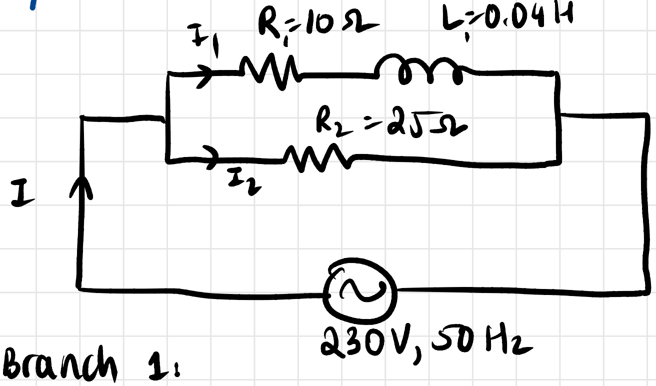
$$I = \frac{V}{Z_T}$$

Admittance

$$Y_1 + Y_2 = Y_T$$

$$I = VY_T$$

Q13. An inductive coil of $R = 10\Omega$, $L = 0.04H$ is connected in parallel with a non-inductive resistor of 25Ω. The combination is connected across 230V, 50Hz. Calculate
(i) current in each branch
(ii) angle of lead/lag
(iii) total current drawn
(iv) phase ange of combination
(v) power factor



Branch 1:

$$Z_1 = R + jX_L \qquad X_L = 4\pi = 12.566$$

$$Z_1 = 10 + j4\pi$$
$$V_1 = 230$$

$$I_1 = \frac{V_1}{Z_1} = 14.32\angle -51.49 \text{ A}$$

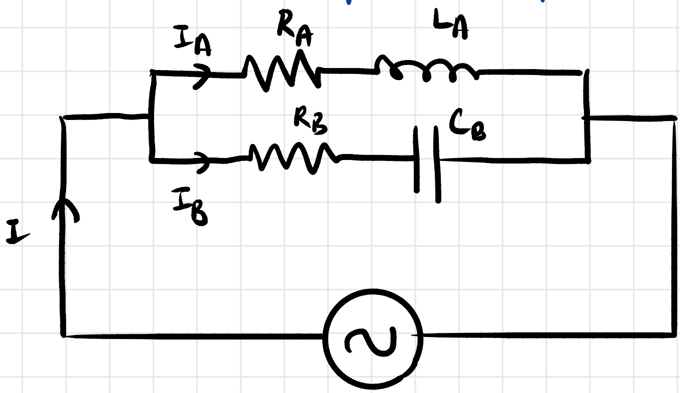
Branch 2

$$Z_2 = R_2 = 25$$
$$I_2 = \frac{230}{25} = 9.2\angle 0 \text{ A}$$

$$I_T = \bar{I}_1 + \bar{I}_2 = 21.30\angle -31.74 \text{ A} \Rightarrow \phi = -31.74$$

$$\cos\phi = 0.85 \text{ lag}$$

Q14. Branch A of a parallel circuit consists of a coil $R = 50\Omega$, $L = 0.1H$. Branch B consists of RC in series, $R = 45\Omega$, $C = 100\mu F$. Calculate the following if $V = 230V$, 50Hz
(i) current (ii) power (iii) power factor



Branch A:

$Z_A = R_A + jX_{LA}$ $\quad$ $X_{LA} = 2\pi f L_A = 10\pi$

$Z_A = 50 + j10\pi$

$I_A = \frac{V}{Z_A} = 3.895\angle -32.142$

Branch B:

$Z_B = R_B - jX_C$ $\quad$ $X_C = \frac{1}{2\pi f C} = \frac{100}{\pi} = 31.83$

$Z_B = 45 - j(31.83)$

$I_B = 4.173\angle 35.274$

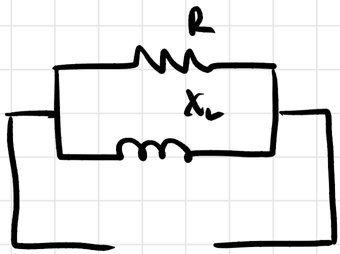
$I_T = I_A + I_B = 6.71\angle 2.88$

$pf = \cos\phi = 0.9987 \Rightarrow$ power $= VI\cos\phi$

$= 958.6$ W lead

Q15. The admittance of a circuit is $(0.05 - j0.08)\mho$, find the value of resistance and inductive reactance of the circuit if the circuit has been connected in (i) parallel (ii) series

(i)

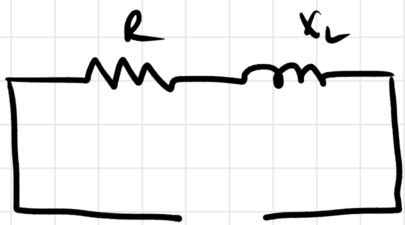


$$Y = (0.05 - j0.08)\mho = G - jB_L$$

$G = 0.05$ $\quad$ $B_L = 0.08$

$R = 20\Omega$ $\quad$ $X_L = 12.5\Omega$

(ii)

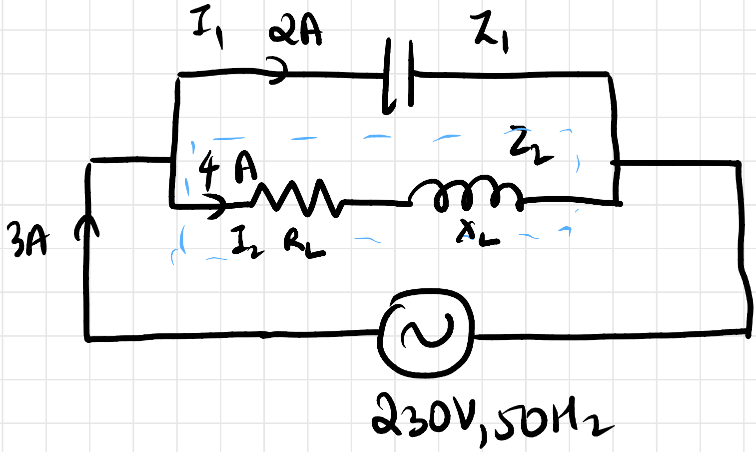


$$Y = (0.05 - j0.08)\mho \Rightarrow Z = 5.62 + j8.99$$

$$Z = R + jX_L$$

$R = 5.62\Omega$ $\quad$ $X_L = 8.99\Omega$

Q.16. A pure capacitance of C Farad is connected in parallel with an inductive coil across 230 V, 50 Hz supply. The current taken by the capacitor is 2A and the current through the inductive coil is 4 A. The total current supplied is 3A.
(a) Find C
(b) Internal resistance of coil, reactance
(c) Phase angle b/w V and I
(d) Phasor



$$Z_1 = X_C = \frac{1}{2\pi f C} \Rightarrow \frac{V}{I} = \frac{1}{2\pi f C} \Rightarrow C = \frac{I}{2\pi f V}$$

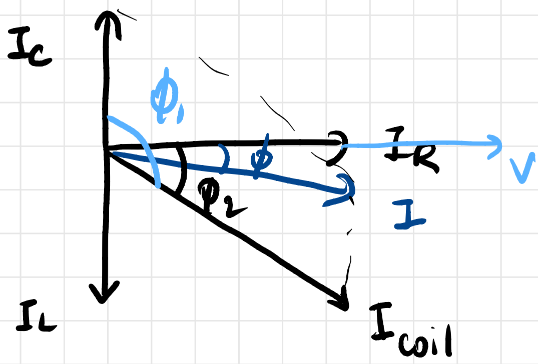
$C = 2.77 \times 10^{-5}$ F $= 27.7\mu F$

$X_C = 115\ \Omega$

$$Z_L = R_L + jX_L$$

$$Z_L = \frac{V}{I_2} = \frac{230}{4} = 57.5$$

Phasor



$$I^2 = I_1^2 + I_2^2 + 2I_1I_2\cos\phi_1$$

$$3^2 = 2^2 + 4^2 + 2\times4\times2\cos\phi_1$$

$$\phi_1 = 133.43$$

$$\varphi_2 = 43.43°$$

$$\cos\phi = \frac{r_{coil}}{Z_{coil}}$$

$$r_{coil} = 41.76\,\Omega$$

$$X_L^2 = Z_L^2 - r_{coil}^2 \Rightarrow X_L = 39.52\,\Omega$$

$$Z_L = 57.5\angle{-43.43}$$
$$Z_C = 115\angle 90$$

$$Y_{tot} = 0.0130\angle 14.47$$

$$Z_{tot} = 76.66\angle{-14.47}$$

$$\phi = -14.47°$$

Q17. Two impendances $(5+j10)$ and $(10+j15)$ are connected in series and the power dissipated in the circuit is 250 W. What will be the power dissipated in the circuit when the same two impedences are connected in parallel across the same source?

$$Z_1 = 5+j10 \qquad Z_2 = 10+j15$$

$$Z_{series} = 15+j25$$

$$P = 250\,W = I^2R = I^2 \times 15$$

$$I^2 = \frac{50}{3} \Rightarrow I = 4.08$$

$$V = I Z_{series} = 119.02\,V$$

$$\phi = -59.04$$

source: 119.02 V

parallel

$$Y_1 = 0.04 - 0.08j \qquad Y_2 = 0.03077 - 0.04615j$$

$$Y_{total} = 0.0708 - 0.126j$$

$$Z_{total} = 3.38 + 6.03j$$

$$I_{total} = \frac{V}{Z_{total}} = 17.216 \angle -60.72$$

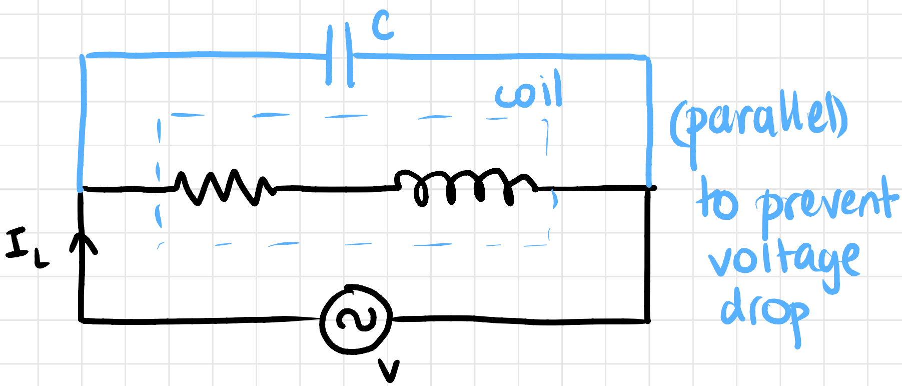
$$\text{power} = I_{total}^2 R = 1001\,W \text{ lag}$$

Power Factor Improvement

The process of making pf close to 1 or making phase angle close to 0.

Reduces power loss

Usually lagging load (coil / inductor)



Phasor